MATH 1650: FACTORING AND DIVISION

EXAMPLE: Find all real zeros of the given polynomial. Check your answer using a graphing utility.

- Solving $p(x) = 2x^3 - x^2 - 20x + 10 = 0$, we factor: $x^2(2x-1) - 10(2x-1) = 0$ or $(2x-1)(x^2-10) = 0$.

  Solving $2x - 1 = 0$ gives $x = \frac{1}{2}$. Solving $x^2 - 10 = 0$ gives $x^2 = 10$ or $x = \pm\sqrt{10}$.

  Graphing $p(x) = 2x^3 - x^2 - 20x + 10$ on desmos shows x-intercepts at $x = 0.5$, and $x = \pm 3.162$.

  Note, $x = \pm 3.162$ are **approximations** of $\pm\sqrt{10}$.

- Solving $f(t) = t^4 - 8t^2 + 15 = 0$, we factor: $\left(t^2 - 5\right)\left(t^2 - 3\right) = 0$.

  Solving $t^2 - 5 = 0$ gives $t = \pm\sqrt{5}$ and solving $t^2 - 3 = 0$ gives $t = \pm\sqrt{3}$.

  Again, desmos gives us **approximations** of these answers: $t = \pm 2.236$ and $t = \pm 1.732$.

EXAMPLE: Perform the following problems using synthetic division. Identify the quotient and the remainder.

- $\left(5x^3 - 2x^2 + 1\right) \div (x - 3)$

  HINT: Since there is no 'x' term, record a '0' in the synthetic division tableu as a placeholder.

  As mentioned in the hint, we enter 0 for the coefficient of x in the dividend.

$$\begin{array}{r|rrrr} 3 & 5 & -2 & 0 & 1 \\ & \downarrow & 15 & 39 & 117 \\ \hline & 5 & 13 & 39 & \boxed{118} \end{array}$$

  Since the dividend was a third degree polynomial function, the quotient is a second degree (quadratic) polynomial function with coefficients 5, 13 and 39: $q(x) = 5x^2 + 13x + 39$. The remainder is $r(x) = 118$.

  Hence, we may write: $5x^3 - 2x^2 + 1 = (x - 3)\left(5x^2 + 13x + 39\right) + 118$.

- $\left(t^3 + 8\right) \div (t + 2)$

  HINT: Record '0' in the synthetic division tableu as placeholders as needed. Also note, $t + 2 = t - (-2)$.

  As the hint suggests, we write $t + 2$ as $t - (-2)$ and enter '0' placeholders for the coefficients of t^2 and t:

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & \downarrow & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & \boxed{0} \end{array}$$

  We get the quotient $q(t) = t^2 - 2t + 4$ and the remainder $r(t) = 0$.

  Relating the dividend, quotient and remainder gives: $t^3 + 8 = (t + 2)\left(t^2 - 2t + 4\right)$.

  You may recognize this result as a specific instance of of the 'sum of cubes' formula.

EXAMPLE: Let $p(x) = 2x^4 + 11x^3 + 11x^2 - 15x - 9$.

- Use the fact that $x = -3$ is a zero of multiplicity 2 to find the remaining real zeros of p.

$$\begin{array}{r|rrrrr}
-3 & 2 & 11 & 11 & -15 & -9 \\
 & \downarrow & -6 & -15 & 12 & 9 \\
\hline
-3 & 2 & 5 & -4 & -3 & \boxed{0} \\
 & \downarrow & -6 & 3 & 3 & \\
\hline
 & 2 & -1 & -1 & \boxed{0} &
\end{array}$$

Hence, $2x^4 + 11x^3 + 11x^2 - 15x - 9 = (x+3)^2\left(2x^2 - x - 1\right)$.

To find the remaining zeros of p, we set $2x^2 - x - 1 = 0$.

Factoring gives $(2x+1)(x-1) = 0$ so the remaining zeros are $x = -\frac{1}{2}$ and $x = 1$.